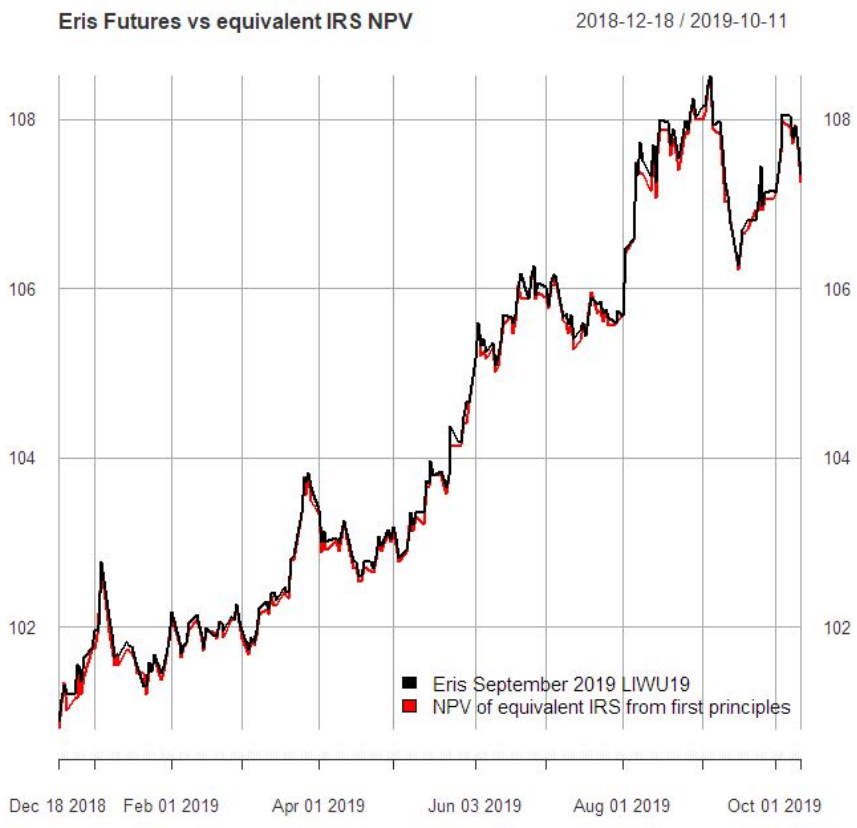
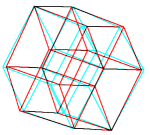
The equivalence of Eris Futures and IRS

- We show from first principles how Eris Futures give you, exactly, the return of an interest rate swap
- A strip of Eurodollar Futures can also be used to construct an Eris Future, but multiple instruments are required and adjustments must be made for discounting and convexity
- In some ways, Eris Swap Futures are analogous to T-note futures, in that they are a long duration instruments quoted on price. We note the similarities, but also the key differences.



This note elucidates how Eris Futures, traded on price, provide equivalent economics to IRS, precisely because they *track IRS total return*. The only relevant difference, for most users of IRS, is the quoting convention - price instead of yield - which is a difference only in optics, not in economics. We will also show that the granularity of Eris Futures products is fine enough that they can be used instead of IRS for most hedging or investment purposes across the yield curve, with all the advantages that futures products bring: concentrated liquidity, standardised contracts, optimised margining (including offsetting vs US Treasury Futures and Eurodollars), and transparent P&L. Only in rare, idiosyncratic cases where exact cash flows must be matched, does an IRS trump Eris



Futures; in all other cases, the returns of interest rate swaps can be achieved using this efficient listed product.

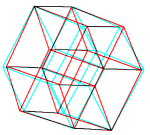
Background

The quoting convention for financial instruments is a complex and interesting topic, permeated as much by the influence of rational design, as by the requirements of expediency and historical inertia. The interest rate swap (IRS) market offers an interesting example of the above. The un/derlying financial instrument of which IRS is a derivative, is of course LIBOR, quoted as a money market yield. It is not surprising then that the designers of IRS, which is meant to swap between fixed and floating interest rate exposures, chose to use *yield* as the quoting convention for the fixed leg, too. This seems reasonable since users will compare the floating leg to the fixed leg, so thought the designers, and it is indeed the case that swaps are often compared across the maturity spectrum, yield against yield, thereby removing the effect of duration. This was an expedient choice, partly driven by marketing concerns, yet the choice was not obvious, and has downsides. While it makes comparing instruments of different maturities easy (this is also the case for bonds, and is why yield is such an important measure within fixed income markets), it has a major disadvantage: there is no direct concept of P&L without requiring further information in the form of indirect (and frequently inaccurate) measures such as duration or pv01.

In futures markets, stock markets, bond markets, and foreign exchange, in other words in most of the world's traded markets, instruments are traded on **price**, even though all of the instruments populating these markets also have varying concepts of yield**.** The reason that price is chosen is that the underlying investor cares about the total return on their investment outlay, which is obviously given by the price of the investment. IRS yields, as they move around, require calculations to get to price, and these calculations, though conceptually straightforward, can be non-trivial in the detail. Eris Futures made the decision to quote their instruments on the total return, in other words, the total present value, in *price*, of an IRS. This is a powerful and very useful choice, which is the same one that was made by the designers of Eurodollar Futures. The latter are not only quoted on price, but every experienced trader will attest to the fact that market talk in Eurodollars is typically about the price of the front contract, and not its yield, strong evidence, if any more were needed, that price is what matters most.

By quoting prices, Eris Futures provide a clean and direct link to returns. Eris Futures go even further than the price, for example of bonds, in that they incorporate directly the net present value of all the cash flows, including the past and presently accruing coupons (something which you need dirty price, in bonds, to get to). The mathematics of yield to price can be complex, but before we proceed, let's make an assertion:

Yield is simply an alternative way to quote price



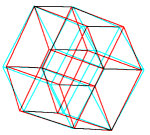
This statement requires a little bit of qualification: for IRS it is true only for "par" instruments, that is, for instruments whose NPV is zero. This is fortuitous because IRS is always quoted in the market as a par instrument, but it's worth keeping in mind that once an IRS's NPV becomes non-zero, then the statement is no longer true, as curve shape comes into the equation. This is why, strictly, generically quoted IRS "yield" is not a yield, but is actually a fixed leg *rate* at which the npv of the IRS would be zero. For our purposes, we will limit ourselves to the par case since that is how IRS is priced (moreover, for an npv-zero IRS, the yield on the fixed leg is equal to the fixed leg rate, which is why the words are interchangeable). As an aside, bond markets which are quoted in yield, such as Japanese Government Bonds or South African Government Bonds, discount all cash flows at the yield to maturity, and therefore the calculation is unambiguous, since there is a monotonic function between yield and price ie: a single yield always maps to a single price.

Eris Swap Futures, being quoted on price, do not suffer from the problem of off-par yield ambiguity, and we will now proceed to show how they are calculated, by going through the steps of pricing the underlying yield-quoted IRS. These steps are exactly what any risk pricing software performs under the hood.

Pricing from first principles

IRS for any given tenor is comprised of two legs of cash flows, the fixed leg, which is quoted in the market as a par coupon rate, and the floating leg, which is derived from the underlying zero curve in the market. The direction of each leg is the opposite of the other; that is, a fixed leg payer, *receives* the floating leg, and vice versa. At inception, the net present value of quoted IRS is zero, meaning the present value of the floating leg, and fixed leg, exactly offset each other. In order to determine the quoted fixed leg rate, we must go through the following steps:

- Obtain a *zero curve* by bootstrapping futures, FRAs, and swap prices in the market. Bootstrapping is a subject that is outside of the scope of this document, and is, because of daycounts, conventions, and interpolation methods, a non-trivial procedure even if the principles are simple. The zero curve allows us to obtain the exact present value (NPV) of any future cash flow, and to interpolate any future *expected* LIBOR rates.
- Take the floating leg convention, in the case of USD IRS, every 3 months, and create cash flows which exactly match the bootstrapped zero curve, that is, they pay the *expected* (*implied*) LIBOR rate on the swap notional for those 3 month periods exactly *even though these future LIBORs are not yet known.* They are determined based on prevailing market rates which in turn determine the zero curve through the bootstrapping process. Given that the floating leg pays exactly the interest rate expected for the period(s) in question, we call this par value. Floating leg pillars are, as



per the name, *floating*. They are not known up front (until 2 business days before the start of the floating rate period), but are *implied* using the zero curve.

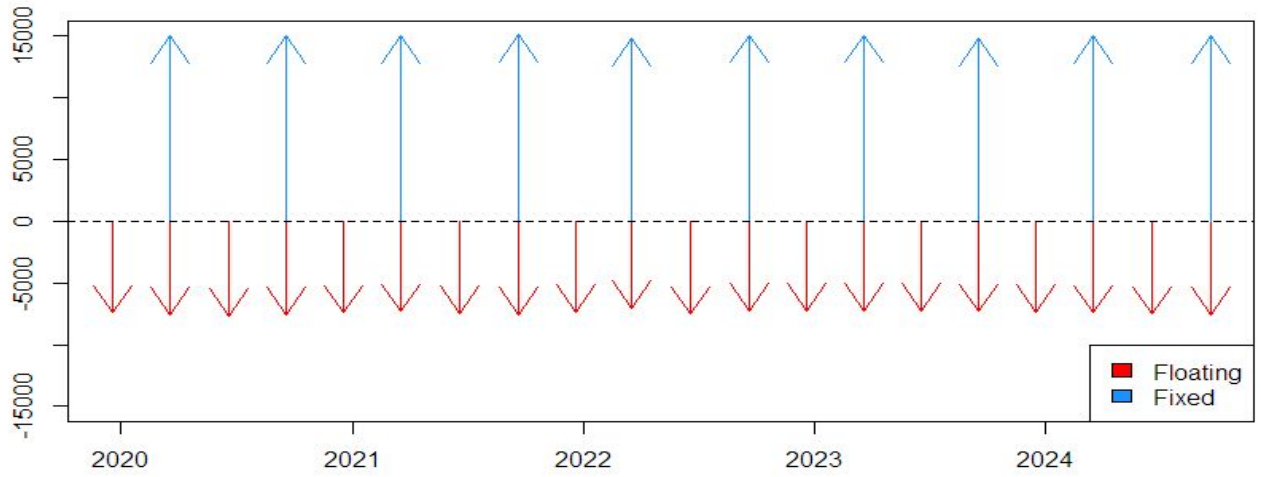
- Take the fixed leg convention, in the case of USD IRS, every 6 months, and create ***identical*** cash flows spaced every 6 months, of size such that the NPV of the fixed cash flows is exactly equal to the NPV of the implied floating leg cash flows, and as such, exactly offsets their value. The fixed leg rate required to obtain these fixed cash flows will become the quoted yield of the IRS. Note the insistence on the word "identical". The fixed leg cash flows are all the same, and never change (other than a few adjustments for weekends or holidays). Neither of these characteristics is true for the floating leg, which instead constantly tracks the zero curve's implied future LIBORs.

We will now price the IRS which underlies the Eris LIWU19 September 2019 5y future, namely a 3% coupon instrument with 6-monthly fixed cash flows, and quarterly (3m LIBOR) floating cash flows. Please note that Eris futures are created up to 9 months before the start date of the underlying IRS, and thus fixed leg coupons may be slightly different to those prevailing when the IRS starts, but as we will see later, this makes little difference to Eris Futures' pricing accuracy. The Eris LIWU19 5y September 2019 Future has a coupon of 3%, which is, within the nearest 25 bps, the coupon which was expected for rates at the time LIWU19 was first listed.

Chart 1 below shows the floating and fixed leg payments for an IRS with a 3% fixed leg, of notional 1m USD matching that of the Eris September 2019 5y Future, as of 30 November 2018.

Chart 1: Cash flows at inception of IRS matching Eris LIWU19 September 2019 Future

Fixed and floating cash flows, represented graphically against a notional of 1m USD, for a 3% 5y IRS with the same dates as that which underlies the Eris 5y LIWU19 future

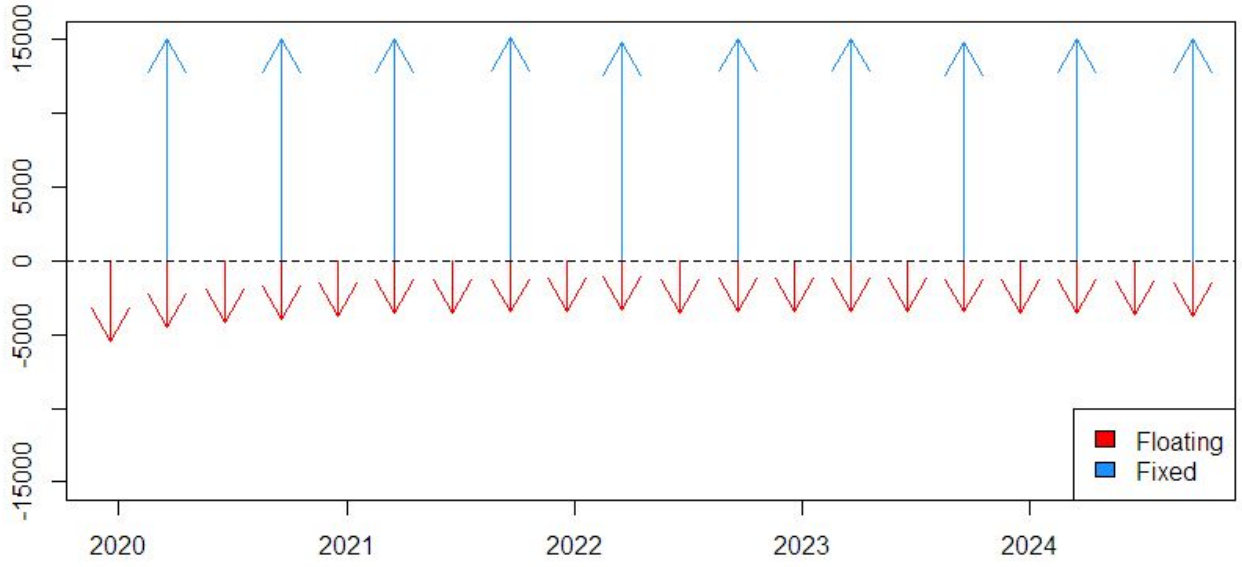


Note how the floating cash flows, double in frequency, appear about half the size of the fixed cash flows. This is what gives us an NPV of close to zero at inception (close to zero as the Eris coupons are set to the nearest 25 basis point increment to the rate that would give a par IRS NPV).

The zero curve, derived as it is from bootstrapping market instruments which move, moves constantly with the market. This is what creates NPV changes for an IRS holder. While the fixed leg payments stay constant, the floating leg payments do not. They fix every time one of the pillar dates is reached, against the floating leg reference instrument, in this case 3m LIBOR, which of course, changes constantly. In Chart 2 below we show how exactly the same IRS, with dates, fixed and floating rate terms matching the Eris September 2019 5y Future, has evolved:

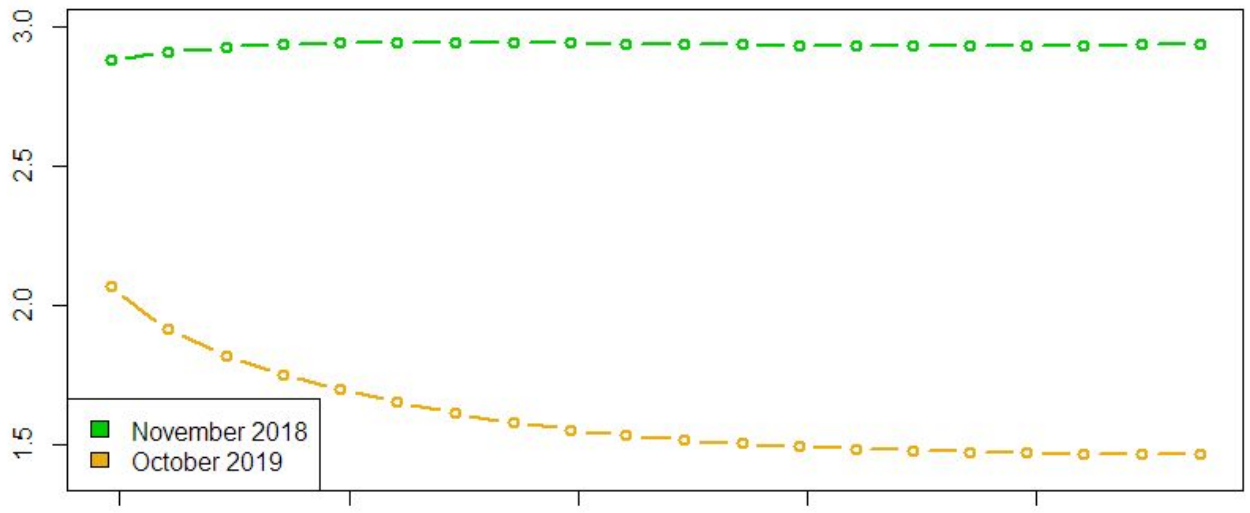
Chart 2: Cashflows at 11 months later for IRS matching Eris LIWU19 September 2019 Future

Fixed and floating cash flows, represented graphically against a notional of 1m USD, for a 3% 5y IRS with the same dates as that which underlies the Eris 5y LIWU19 future, but priced off the zero curve as of 10 October 2019



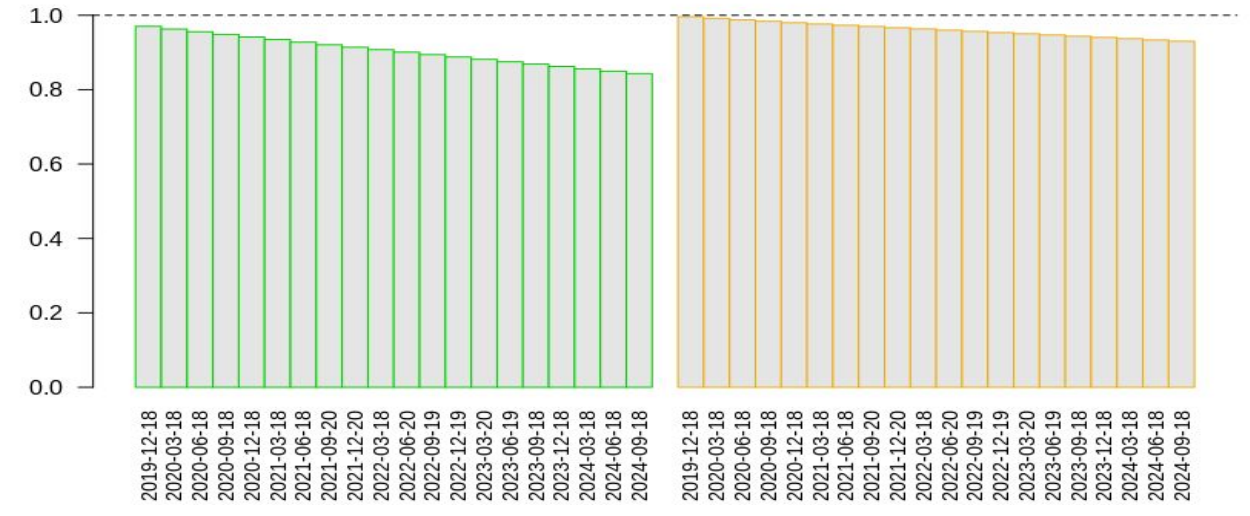
Note how the blue, fixed pillars are identical to those in Chart 1, yet the floating pillars have reduced dramatically in size, because of the significant rally in interest rates in the past year. This will lead to a significant NPV change. In order to illustrate the reason for these floating leg changes, note the zero curves for the two dates in question in Chart 3 below:

Chart 3: Implied zero curves for 30 November 2018 (when LIWU19 was listed) and 10 October 2019



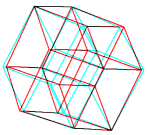
Discount Factors are what are used to price a future cash flow off the zero curve, by using them as multiplicative coefficients. They are derived by continuously compounding the zero-curve interest rate for the corresponding pillar date. Discount factors related to the above zero curve are shown in chart 4, below, for the same yield curves as in Chart 3.

Chart 4: Discount factors corresponding to Eris Futures floating pillars, based on the zero curves in Chart 3



[1]

[1] Green: 30 November 2018, Yellow: 10 October 2019



Note how higher yields (green - corresponding to 30 November 2018 market conditions) result in lower discount factors, meaning that when multiplied by the cash flow, they will reduce its NPV further than the discount factors associated with lower yield (yellow - corresponding to the 10 October 2019 market conditions).

The fixed leg and the floating leg are discounted off exactly the same zero curve, with the difference that while the floating leg cash flows change to reflect the zero curve, the fixed leg cash flows are fixed at inception. Lower rates benefit an IRS receiver in two ways: they have lower cash flows to pay in the future on the floating leg, and the fixed leg is discounted at lower rates (multiplied by higher discount factors), for a higher NPV. The converse is obviously also true for higher rates or IRS payers.

The tables on the following two pages give a breakdown of exactly how an IRS corresponding to the Eris LIWU19 Future was priced on 30 November 2018, and on 10 October 2019:

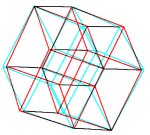
Table 1: Eris Sep 19-equivalent IRS pricing as of 30 November 2018

FIXED LEG

Payment Date	Accrual Days	Rate	Amount	Discount Factor	Zero Rate	NPV
2020-03-18	180	0.03	15,000.00	0.963	2.911	14,444
2020-09-18	180	0.03	15,000.00	0.948	2.939	14,227
2021-03-18	180	0.03	15,000.00	0.935	2.945	14,019
2021-09-20	182	0.03	15,166.67	0.921	2.945	13,964
2022-03-18	178	0.03	14,833.33	0.908	2.942	13,463
2022-09-19	181	0.03	15,083.33	0.894	2.938	13,489
2023-03-20	181	0.03	15,083.33	0.881	2.935	13,294
2023-09-18	178	0.03	14,833.33	0.869	2.934	12,885
2024-03-18	180	0.03	15,000.00	0.856	2.936	12,839
2024-09-18	180	0.03	15,000.00	0.843	2.941	12,647
					TOTAL:	**135,271**

FLOATING LEG

Payment Date	Accrual Days	Rate	Amount	Discount Factor	Zero Rate	NPV
2019-12-18	91	0.03	7,414.98	0.970	2.882	7,194
2020-03-18	91	0.03	7,568.64	0.963	2.911	7,288
2020-06-18	92	0.03	7,651.32	0.956	2.930	7,312
2020-09-18	92	0.03	7,554.06	0.948	2.939	7,165



2020-12-18	91	0.03	7,425.07	0.941	2.944	6,990
2021-03-18	90	0.03	7,314.71	0.935	2.945	6,836
2021-06-18	92	0.03	7,455.59	0.928	2.945	6,917
2021-09-20	94	0.03	7,597.50	0.921	2.945	6,995
2021-12-20	91	0.03	7,336.20	0.914	2.944	6,705
2022-03-18	88	0.03	7,066.07	0.908	2.942	6,413
2022-06-20	94	0.03	7,526.94	0.901	2.940	6,780
2022-09-19	91	0.03	7,282.54	0.894	2.938	6,513
2022-12-19	91	0.03	7,297.60	0.888	2.936	6,479
2023-03-20	91	0.03	7,298.45	0.881	2.935	6,433
2023-06-19	91	0.03	7,301.22	0.875	2.934	6,389
2023-09-18	91	0.03	7,325.25	0.869	2.934	6,363
2023-12-18	91	0.03	7,370.15	0.862	2.935	6,355
2024-03-18	91	0.03	7,412.04	0.856	2.936	6,344
2024-06-18	92	0.03	7,529.38	0.850	2.938	6,397
2024-09-18	92	0.03	7,567.44	0.843	2.941	6,381
					TOTAL:	**134,248**
					NPV	**1,023**

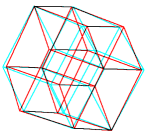
Table 2: Eris Sep 19-equivalent IRS pricing as of 10 October 2019

FIXED LEG

Payment Date	Accrual Days	Rate	Amount	Discount Factor	Zero Rate	NPV
2020-03-18	180	0.03	15,000.00	0.992	1.912	14,875
2020-09-18	180	0.03	15,000.00	0.984	1.749	14,755
2021-03-18	180	0.03	15,000.00	0.977	1.652	14,648
2021-09-20	182	0.03	15,166.67	0.970	1.577	14,708
2022-03-18	178	0.03	14,833.33	0.963	1.533	14,290
2022-09-19	181	0.03	15,083.33	0.957	1.503	14,431
2023-03-20	181	0.03	15,083.33	0.950	1.484	14,332
2023-09-18	178	0.03	14,833.33	0.944	1.472	13,998
2024-03-18	180	0.03	15,000.00	0.937	1.466	14,055
2024-09-18	180	0.03	15,000.00	0.930	1.465	13,953
					TOTAL:	**144,045**

FLOATING LEG

Payment Date	Accrual Days	Rate	Amount	Discount Factor	Zero Rate	NPV
2019-12-18	91	0.02	5,422.41	0.996	2.067	5,401
2020-03-18	91	0.02	4,474.62	0.992	1.912	4,437

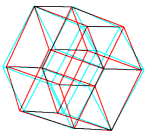


2020-06-18	92	0.02	4,149.70	0.988	1.816	4,098
2020-09-18	92	0.02	3,945.74	0.984	1.749	3,881
2020-12-18	91	0.01	3,759.37	0.980	1.699	3,684
2021-03-18	90	0.01	3,522.86	0.977	1.652	3,440
2021-06-18	92	0.01	3,496.22	0.973	1.612	3,402
2021-09-20	94	0.01	3,478.62	0.970	1.577	3,374
2021-12-20	91	0.01	3,389.70	0.967	1.552	3,276
2022-03-18	88	0.01	3,276.66	0.963	1.533	3,157
2022-06-20	94	0.01	3,504.69	0.960	1.516	3,364
2022-09-19	91	0.01	3,406.26	0.957	1.503	3,259
2022-12-19	91	0.01	3,421.18	0.953	1.493	3,262
2023-03-20	91	0.01	3,425.55	0.950	1.484	3,255
2023-06-19	91	0.01	3,445.05	0.947	1.477	3,262
2023-09-18	91	0.01	3,481.35	0.944	1.472	3,285
2023-12-18	91	0.01	3,523.39	0.940	1.468	3,313
2024-03-18	91	0.01	3,550.47	0.937	1.466	3,327
2024-06-18	92	0.01	3,634.51	0.934	1.465	3,393
2024-09-18	92	0.01	3,699.55	0.930	1.465	3,441
					TOTAL:	**71,314**
					NPV	**72,732**

In all cases, the *Amount* column is multiplied by the *Discount Factor* column to give the NPV. The value of the IRS is equal to the sum of fixed cash flow NPVs, less the sum of the floating cash flow NPVs (for an IRS receiver ie an Eris September 2019 Future *buyer*). Note how in November 2018 the floating and fixed leg NPVs almost exactly offset each other, while in October 2019, the NPV of a 1m notional Eris September 2019 has grown to 72 thousand dollars, or approximately 7% of the notional. Unsurprisingly, the Eris Sep 2019 5y Future was trading at 107.2 at the same time, demonstrating clearly how Eris exactly matches the P&L on an IRS.

Price Alignment Interest ("PAI")

In addition to accurately tracking the valuation of the IRS underlying each Eris Futures product, the *variation margin interest that accrues in the case of an IRS* is also taken into account within the Eris Futures price. Derivatives typically require an initial margin, but when marked to market every day, the resulting P&L results in a *variation margin*, which must bear interest, crediting the party posting the *variation margin*, debiting the party receiving the *variation margin*. For example, in a scenario where rates fall, earlier interest rate swaps will have positive NPV, and that NPV change results in a *variation margin* payment equal to the NPV, which must be made by the fixed-leg payer party. This running balance earns interest



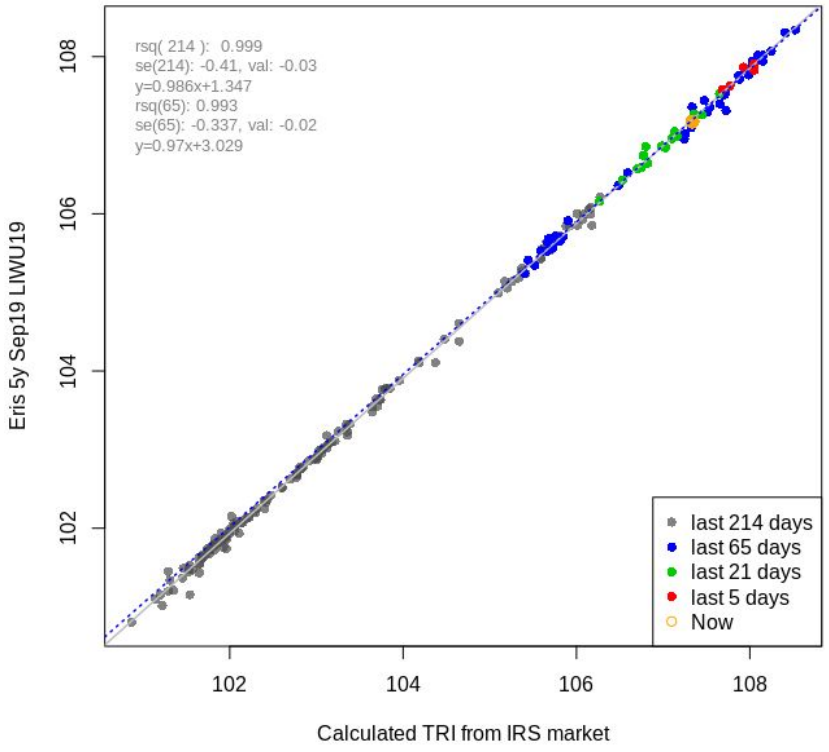
daily at the overnight fed funds rate, and is part of the economics of trading IRS. The direction of PAI is the opposite of the direction of cumulative P&L, since it is the losing party which must post collateral, and this collateral earns interest. Eris Futures track the interest paid on variation margin though the Price Alignment Interest ("PAI"), in which the underlying IRS's NPV's interest rate economics are added (or subtracted in the case of NPV negative) to the futures contract price. In this way, Eris Futures account for ALL the economics of IRS, even this relatively small, but important, total return constituent.

Eris Accuracy

In Chart 5 below, we have performed the above calculations for every day up to an including 10 October 2019, and compared the resulting first principles pricing, to that of the corresponding September 2019 LIWU19 Eris Future. As is clear, the regression is exceedingly tight, with but a small, sub 10c standard error (attributable to differences in mark-to-market source and time of day for our calculations).

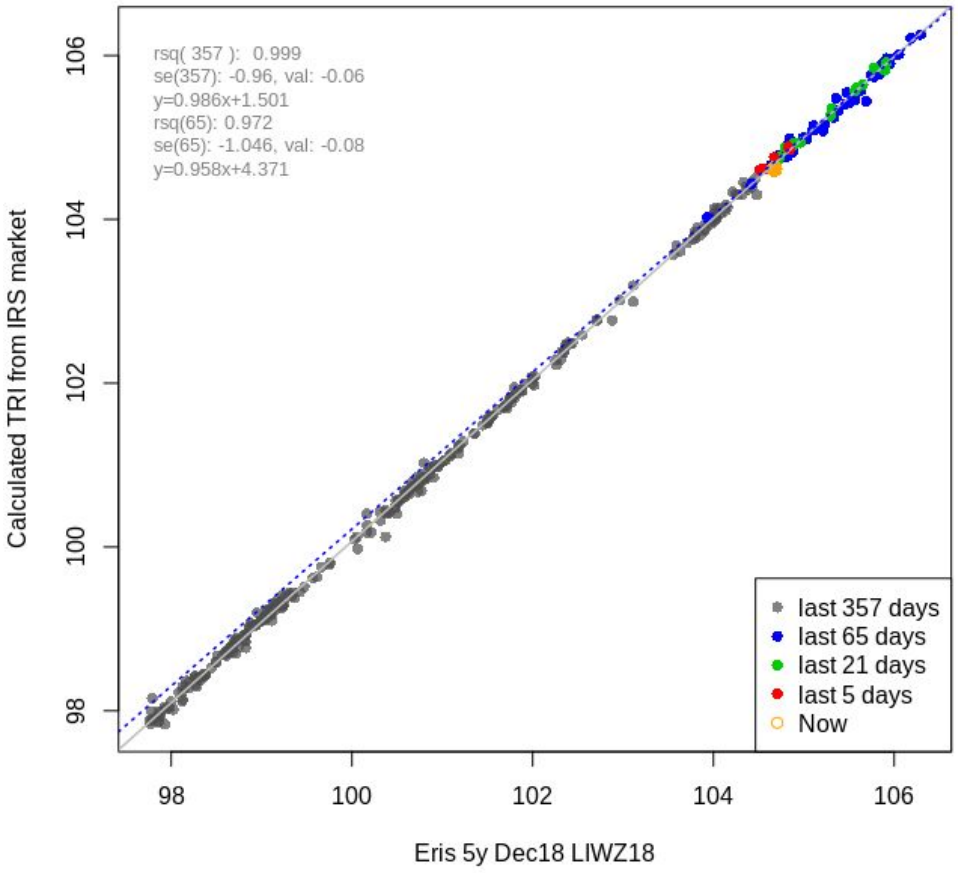
This shows the extent to which the P&L on an Eris Future is essentially identical to that of the equivalent underlying IRS. Since new Eris Futures are created every 3 months, for tenors 2y, 3y, 4y, 5y, 7y, 10y, 20y and 30y, a broad range of accurate swap market exposures along the yield curve are possible.

Chart 5 : Regression of LIWU19, Eris September 2019 future, and first-principles pricing of the equivalent underlying IRS, at prevailing market rates



In order to demonstrate that Eris pricing accuracy further, we have performed the same analysis as above, but for the Eris 5y LIWZ18 December 2018 future, which has a 2.75% coupon, and which, unlike LIWU19, has already experienced some cashflows.

Chart 6 : Regression of LIWZ18, Eris Dec 2018 future, and first-principles pricing of the equivalent underlying IRS, at prevailing market rates



Finally in order to demonstrate that Eris Futures may be used instead of IRS even if the Eris contract fixed rate and dates do not match that of one's at-market par value or custom IRS, we show in chart 7 and table 3 below, some regression diagnostics for the Eris LIWU19 September 2019 5y Future versus a number of total return IRS indices that we have created, representing a typical IRS. These have varying effective dates (Jan 2019, September 2019, January 2021), and different coupons (1%, 2%, 3%, 4%, and 5%).

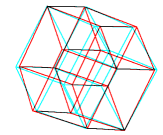
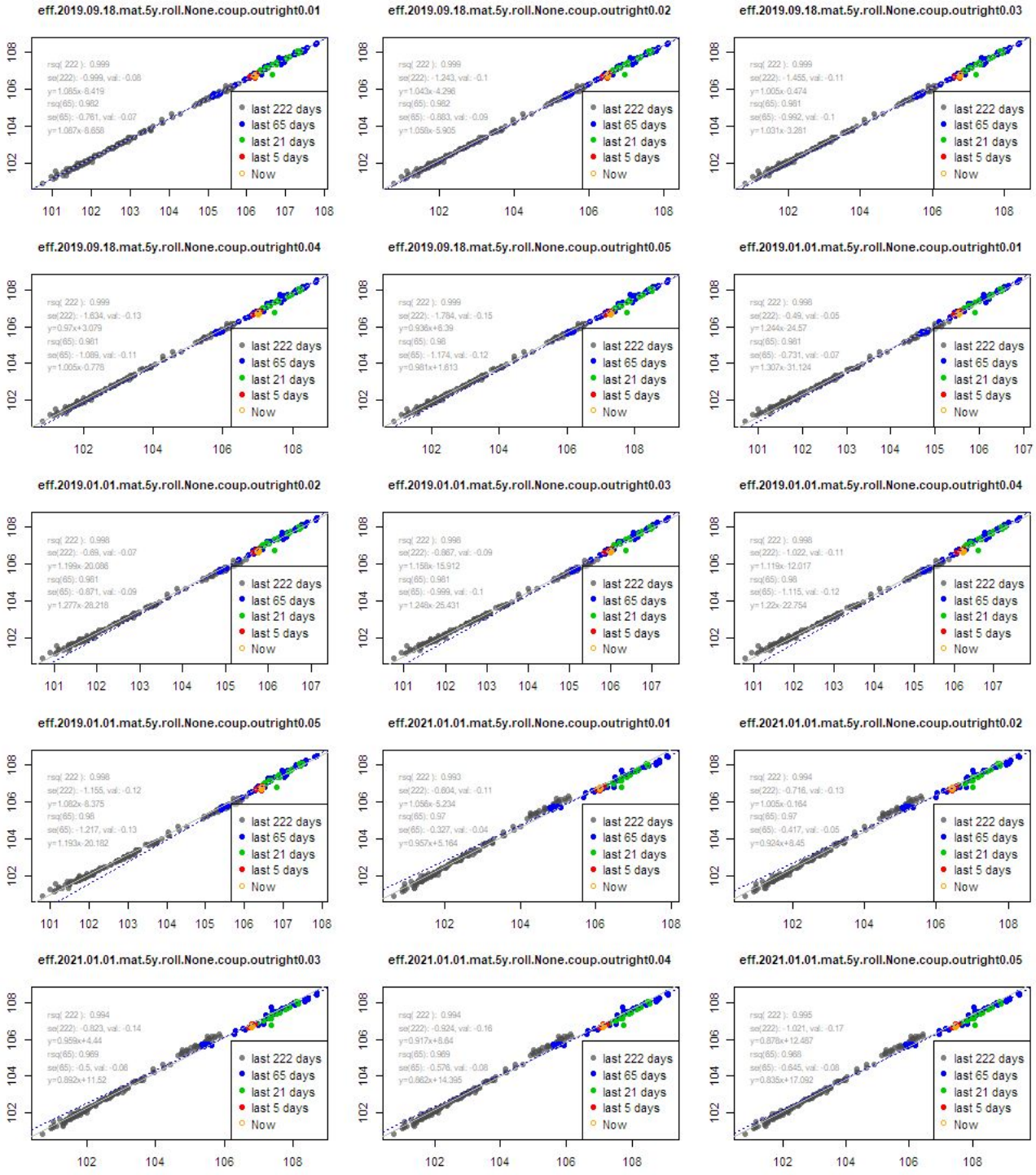


Chart 7 : Comparison regressions for multiple 5y total return indices, of varying start dates and coupons, with the Eris LIWU19 September 2019 5y future



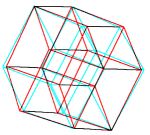


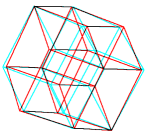
Table 3: Regression diagnostics

eff.2019.09.18.mat.5y.roll.None.coup.outright0.01 se: 0.08 rsq: 0.9988 beta: 1.08
eff.2019.09.18.mat.5y.roll.None.coup.outright0.02 se: 0.08 rsq: 0.9988 beta: 1.04
eff.2019.09.18.mat.5y.roll.None.coup.outright0.03 se: 0.08 rsq: 0.9988 beta: 1
eff.2019.09.18.mat.5y.roll.None.coup.outright0.04 se: 0.08 rsq: 0.9988 beta: 0.97
eff.2019.09.18.mat.5y.roll.None.coup.outright0.05 se: 0.08 rsq: 0.9987 beta: 0.94
eff.2019.01.01.mat.5y.roll.None.coup.outright0.01 se: 0.1 rsq: 0.9981 beta: 1.24
eff.2019.01.01.mat.5y.roll.None.coup.outright0.02 se: 0.1 rsq: 0.998 beta: 1.2
eff.2019.01.01.mat.5y.roll.None.coup.outright0.03 se: 0.1 rsq: 0.998 beta: 1.16
eff.2019.01.01.mat.5y.roll.None.coup.outright0.04 se: 0.1 rsq: 0.9979 beta: 1.12
eff.2019.01.01.mat.5y.roll.None.coup.outright0.05 se: 0.11 rsq: 0.9978 beta: 1.08
eff.2021.01.01.mat.5y.roll.None.coup.outright0.01 se: 0.19 rsq: 0.9932 beta: 1.05
eff.2021.01.01.mat.5y.roll.None.coup.outright0.02 se: 0.18 rsq: 0.9937 beta: 1
eff.2021.01.01.mat.5y.roll.None.coup.outright0.03 se: 0.18 rsq: 0.994 beta: 0.96
eff.2021.01.01.mat.5y.roll.None.coup.outright0.04 se: 0.17 rsq: 0.9943 beta: 0.91
eff.2021.01.01.mat.5y.roll.None.coup.outright0.05 se: 0.17 rsq: 0.9946 beta: 0.88

The standard error shows, in dollars, the mismatch of an Eris contract versus a “pure” total return index made up as per its name (eff: effective data, mat: maturity, roll: roll period - always None meaning never rolled, coup: coupon, in these cases outrights 1% through 5%). It is clear from the images and the rsq (r squared) figures that Eris is exceedingly accurate. Standard errors below 0.1 (10 cents) are essentially attributable to small differences in timing of market data for calculations between our TRIs and Eris Futures.

Clearly, when beta diverges significantly, that is, when Eris has significantly different duration to the underlying TRI, tracking error increases modestly. What is obvious though, is that even for quite different coupons, and quite different starting periods, Eris does an excellent job of matching 5y IRS returns. Note that we have not included other maturity IRS, but these would be similarly tracked by the equivalent Eris maturity. For hedging purposes, and in order to adjust for the beta of Eris vs IRS, one would match the DV01 of the IRS to the DV01 of an Eris position. For example:

Target IRS DV01 per 100k dollars: 55
LIWU19 DV01: 49
Target notional: 10 million



Eris contract notional: 100000
*Eris LIWU19 contracts to do: (10000000 / 100000) * 55/49 = 112.24*

Please note that the above is an example only; readers should ensure numbers are refreshed and accurate.

Eris vs Eurodollars

In the first installment of this series of Eris trade notes we established that Eris Futures are the superior instrument when the objective is to track IRS, and Eurodollar Futures are the superior instrument for fine-grained exposure to the short-end term structure evolution. Yet both products' ultimate underlying is a 3m LIBOR rate, and therefore Eris Futures, and by extension IRS, can be built accurately out of Eurodollars. The issue is that this would require numerous Eurodollar contracts, with enough different notional sizes for each pillar date that it would be very impractical. Instead the best approach is a statistical one, making use of software such as R's regression tools or the Microsoft Excel regression functions ("SLOPE", "INTERCEPT", "RSQ"). Usually two Eurodollar futures will be sufficient to obtain a high r-squared, as can be seen in the next two tables:

Table 4: R-squared obtained when hedging Eris 5y September 2019 with a single Eurodollar Future.

EDZ19 0.822
EDH20 0.904
EDM20 0.933
EDU20 0.951
EDZ20 0.962
EDH21 0.975
EDM21 0.982
EDU21 0.986
EDZ21 0.988
EDH22 0.985
EDM22 0.983
EDU22 0.978
EDZ22 0.971
EDH23 0.963
EDM23 0.952
EDU23 0.940
EDZ23 0.926
EDH24 0.914
EDM24 0.901
EDU24 0.888
EDZ24 0.870

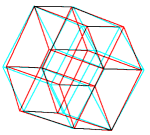


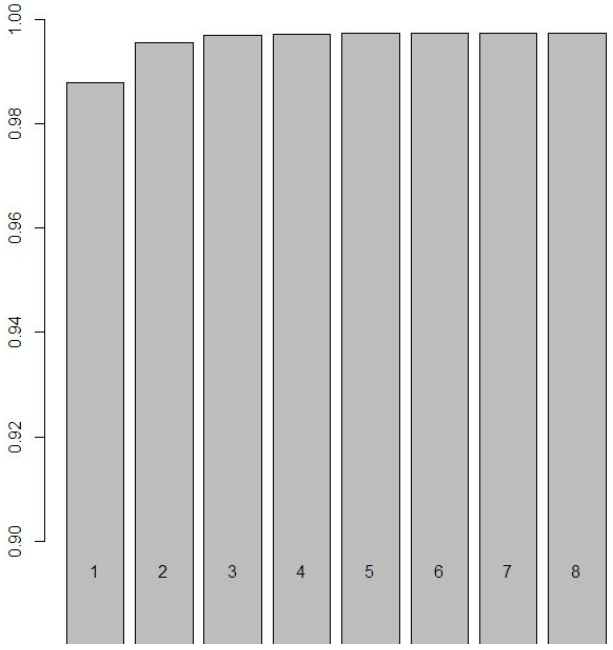
Table 4 shows that the EDZ21, that is the December 2021 Eurodollar future, was the best hedge for Eris LIWU19 with an r-squared of 0.988, and that earlier or later contracts progressively deteriorate in hedge quality. This is unsurprising, as EDZ21 has a maturity roughly halfway through the tenor of the 5y LIWU19. A single futures hedge will only take into account market directionality, however, and not yield curve slope changes. For that we will need two futures:

Table 5: R-squared obtained when hedging Eris 5y September 2019 with two Eurodollar Futures

	EDZ19	EDH20	EDM20	EDU20	EDZ20	EDH21	EDM21	EDU21	EDZ21	EDH22	EDM22	EDU22	EDZ22	EDH23	EDM23	EDU23	EDZ23	EDH24	EDM24	EDU24	EDZ24
EDZ19	0.822	0.915	0.937	0.953	0.963	0.975	0.982	0.987	0.990	0.991	0.992	0.993	0.992	0.990	0.987	0.985	0.981	0.978	0.974	0.969	0.966
EDH20	0.915	0.904	0.938	0.954	0.964	0.975	0.982	0.987	0.991	0.991	0.993	0.995	0.995	0.995	0.994	0.992	0.990	0.989	0.986	0.984	0.982
EDM20	0.937	0.938	0.933	0.956	0.965	0.976	0.982	0.986	0.990	0.991	0.993	0.995	0.996	0.996	0.995	0.994	0.993	0.991	0.990	0.988	0.987
EDU20	0.953	0.954	0.956	0.951	0.965	0.977	0.982	0.986	0.990	0.990	0.992	0.994	0.995	0.995	0.995	0.995	0.994	0.993	0.992	0.991	0.990
EDZ20	0.963	0.964	0.965	0.965	0.962	0.978	0.983	0.986	0.989	0.990	0.992	0.993	0.994	0.995	0.995	0.995	0.994	0.994	0.993	0.992	0.991
EDH21	0.975	0.975	0.976	0.977	0.978	0.975	0.983	0.986	0.989	0.989	0.991	0.993	0.994	0.994	0.994	0.994	0.994	0.994	0.994	0.993	0.993
EDM21	0.982	0.982	0.982	0.982	0.983	0.983	0.982	0.986	0.989	0.989	0.990	0.991	0.992	0.992	0.993	0.993	0.993	0.994	0.994	0.993	0.993
EDU21	0.987	0.987	0.986	0.986	0.986	0.986	0.986	0.986	0.988	0.988	0.989	0.990	0.990	0.991	0.991	0.992	0.992	0.992	0.992	0.992	0.992
EDZ21	0.990	0.991	0.990	0.990	0.989	0.989	0.989	0.988	0.988	0.988	0.988	0.989	0.989	0.989	0.989	0.990	0.990	0.990	0.991	0.991	0.991
EDH22	0.991	0.991	0.991	0.990	0.990	0.989	0.989	0.988	0.988	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.985	0.986	0.986	0.986	0.986
EDM22	0.992	0.993	0.993	0.992	0.992	0.991	0.990	0.989	0.988	0.985	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983	0.983
EDU22	0.993	0.995	0.995	0.994	0.993	0.993	0.991	0.990	0.989	0.985	0.983	0.978	0.979	0.980	0.980	0.980	0.980	0.979	0.979	0.979	0.979
EDZ22	0.992	0.995	0.996	0.995	0.994	0.994	0.992	0.990	0.989	0.985	0.983	0.979	0.971	0.972	0.973	0.973	0.974	0.974	0.973	0.973	0.973
EDH23	0.990	0.995	0.996	0.995	0.995	0.994	0.992	0.991	0.989	0.985	0.983	0.980	0.972	0.963	0.967	0.968	0.968	0.968	0.968	0.968	0.968
EDM23	0.987	0.994	0.995	0.995	0.995	0.994	0.993	0.991	0.989	0.985	0.983	0.980	0.973	0.967	0.952	0.956	0.958	0.960	0.961	0.962	0.961
EDU23	0.985	0.992	0.994	0.995	0.995	0.994	0.993	0.992	0.990	0.985	0.983	0.980	0.973	0.968	0.956	0.940	0.948	0.950	0.954	0.957	0.957
EDZ23	0.981	0.990	0.993	0.994	0.994	0.994	0.993	0.992	0.990	0.985	0.983	0.980	0.974	0.968	0.958	0.948	0.926	0.931	0.936	0.942	0.948
EDH24	0.978	0.989	0.991	0.993	0.994	0.994	0.994	0.992	0.990	0.986	0.983	0.979	0.974	0.968	0.960	0.950	0.931	0.914	0.922	0.932	0.936
EDM24	0.974	0.986	0.990	0.992	0.993	0.994	0.994	0.992	0.991	0.986	0.983	0.979	0.973	0.968	0.961	0.954	0.936	0.922	0.901	0.918	0.918
EDU24	0.969	0.984	0.988	0.991	0.992	0.993	0.993	0.992	0.991	0.986	0.983	0.979	0.973	0.968	0.962	0.957	0.942	0.932	0.918	0.884	0.892
EDZ24	0.966	0.982	0.987	0.990	0.991	0.993	0.993	0.992	0.991	0.986	0.983	0.979	0.973	0.968	0.961	0.957	0.948	0.936	0.918	0.892	0.870

Owing to the large amount of data, we have had to use a screen grab, but we have been able to increase the r-squared from 0.988 for a single-instrument hedge, to 0.996, using a combination of EDM20 (June 2020,) and EDZ22 (Dec 2022) or EDH23 (March 2023). Using the same principles, we can go to three dimensions, thereby hedging curvature too, but this is often unnecessary as the r-squared improves only marginally, in our case to 0.997 by using EDH0 (March 20), EDZ1 (Dec 2021), and EDH4 (March 2024). In fact, as can be seen below in chart 8, going to more than three Eurodollar contracts is not necessary, as the regression does not essentially improve. Nevertheless, managing three futures is more cumbersome than managing a single one, and what is not able to be shown here, is the convexity factor which Eris takes into account, since there is not enough data during a significantly volatile period for convexity to be noticable. However as we explained in depth in the first installment of this series, while Eurodollar Futures are effective interest rate hedges, their lack of convexity can be a drawback when hedging accuracy *during volatile periods* is desired. Eris Futures, by contrast, perfectly match the convexity of their underlying IRS.

Chart 8: Regression r-squared by number of Eurodollar Futures used to replicate Eris 5y September 2019 LIWU19.

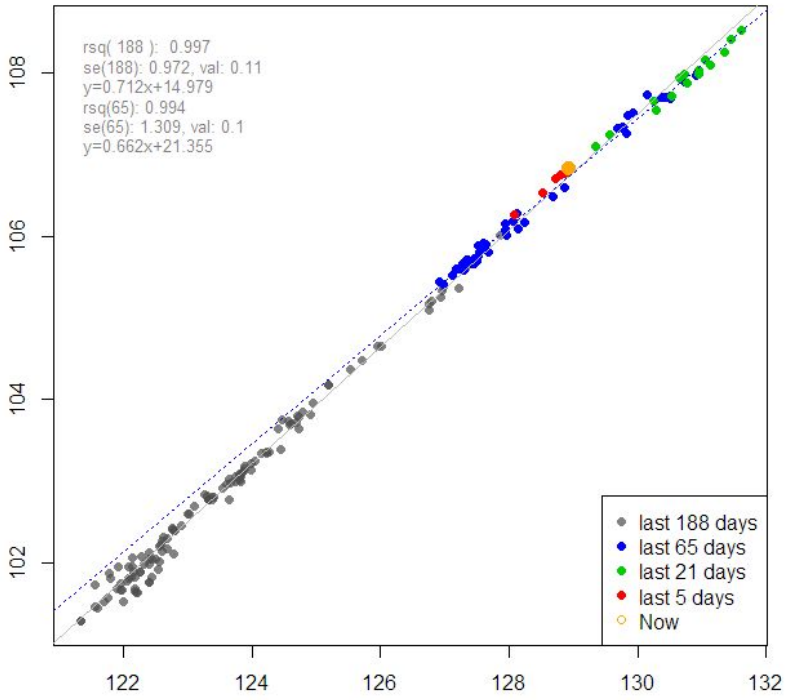


Please contact thomas@crvm.io for further information on how to obtain best hedging.

T-Note comparison

Like Eris Futures, Treasury Futures reference a longer duration instrument, and the 10y Note Future references the cheapest to deliver of the eligible delivery basket. Both being longer dated instruments, it is not surprising that the regression of Eris 5y September 2019 and the T-note future is high, as can be seen in Chart 9 below:

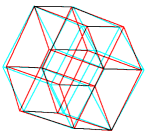
Chart 9: Regression of Eris 5y September 2019 LIWU9, with TYU9 during 2019



Though the regression has a fairly high r-squared of 0.972, this is probably not quite good enough for hedging purposes, and we also note a key difference between the T-note future and the Eris-IRS regressions above: the line's y intercept is very far from the origin (15 points). This is largely because of the difference between the Eris 3% coupon and the Treasury Note Future's 6% implied coupon. Additionally, one must consider the basis between the LIBOR curve and the Treasury yield curve, meaning the *spread* between the two (also known as the asset *swap spread*). This spread can vary, causing P&L, and this is visible in the bottom left of the chart, where hedging accuracy deteriorates. We will be exploring swap spread trading in a future edition of this series.

Summary

We have shown from first principles that Eris Futures very accurately track the P&L of an underlying IRS, and that the (par) swap rate and an Eris price are essentially two different ways of tracking the same thing. We further showed that as the Eris price is essentially the total return of a swap, it is a better measure of analysing long value of a swap than comparing the change of a par swap rate. And lastly, we prove that even with different coupons and start dates, Eris Futures provide equivalent market exposure and substantially



the same economic returns as an IRS, proving the initial assertion made, that for most users of IRS, Eris Futures provide an efficient alternative to over the counter swaps.
